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Pre-service Secondary Mathematics Teachers' Definitions of Mathematics Terms in their Video-Lesson Presentations: A Deductive Content Analysis

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Abstract

Taking into account the nature of mathematics as an exact science and the essential role of mathematics teachers' knowledge of fundamental mathematics definitions, the authors investigated the pre-service secondary mathematics teachers (PSMTs) knowledge of a good mathematics definition in their group video-lesson presentations related to algebra, geometry, descriptive statistics, and number theory. The quality of their definitions can provide a glimpse of their preparedness to teach mathematics at the secondary level. Drawing on Leikin and Zazkis' (2010) framework to analyze teacher-generated definitions, and Borasi's (1992) characterizations of a good definition, the authors developed an analytical framework to analyze a total 109 definitions from 90 different mathematical terms. Results reveal that 57 or 52 % of the definitions were weak and suggest PSMTs lack of precision needed in stating definitions of mathematical terms. This could be attributed to PSMTs' lack of knowledge about the characteristics of a good definition of a mathematical term, and lack of rigor in the use of English language to clearly express the precise meaning of their definitions. The authors recommend PSMTs to be exposed more to activities that would develop their skill in defining. Follow-up studies are also recommended that would further guide mathematics educators in designing intervention programs for the development and improvement of PSMTs' skills in crafting good mathematical definitions.

Key words: Mathematics content knowledge, mathematics definitions, student-created videos, pre-service secondary mathematics teachers, video-lesson presentation

1 Introduction

The rapid advances and innovations in information and communication technologies (ICTs) have brought about significant changes in human interaction and learning [15, 28]. Students are becoming increasingly hooked with technology. To supplement learning, they can view video-lessons, podcasts, and interactive online applications. Hence, to be relevant, educators need to update their teaching methods by considering active learning approach by seeing the need to integrate the use of technology in their lessons to increase student engagement and learning.

With an increased emphasis on active learning approach in the mathematics teaching and learning [2, 10, 20, 21], assigning students individually or in small groups to come up with video-lesson presentations has been one of the methods teachers consider [2, 10, 18, 20, 21, 33, 35, 40, 48]. This method of student-created video-lesson production and presentation provides a space for students to hone their communication and presentation skills while providing them opportunity to reflect on their learning of the mathematical concepts and ideas they are presenting [20].

Recent studies about student-created videos have focused on creating video lessons that focus on providing worked examples with explanations to improve students' achievement test scores [21], problem-solving skill [20, 21], communication and teamwork skills [20], student engagement [10], and processes of designing and development [10]. However, there seems to be not enough attention given to the correctness of the definitions of mathematical terms used in the video-lesson presentations.Precision in stating mathematical definitions and theorems should not be taken lightly since they are powerful tools that can influence one's ability to effectively communicate mathematical ideas. Through clear and precise mathematical definitions, students can organize and understand mathematical ideas which they need to be able to communicate, reason and prove effectively.

Mathematics teachers should have a strong background in mathematics and that their ability and expertise, as well, must be reflected in the ways they communicate mathematical ideas in the classroom which should include video lessons. Mathematics teachers should, therefore, be well-motivated to be able to clearly and correctly craft good mathematical definitions. In this study, mathematical definition is defined as a description of the properties of a mathematical object (e.g., function, circle, triangle) and the relations among those properties, and the disciplinary practice of definition construction refers to mathematical defining. The need for mathematics teachers to develop the ability to clearly and correctly craft good mathematical definitions should begin while they are still students in a teacher-training institution. Mathematics courses for pre-service teachers should provide space for the PSMTs to practice mathematical defining and examining the correctness of the definitions of mathematical terms. Attention should be given to investigating the quality of PSMTs' definitions of mathematical terms in their mathematics courses, and during practice teaching and demo-teaching.

Therefore, the aim of the present study is to examine the correctness of the PSMT definitions of mathematical terms in their video-lesson presentations. The quality of PSMTs' mathematics definitions in their video-lesson presentations can provide a glimpse of the PSMTs' preparedness for mathematics teaching. In this study the authors developed a framework to analyze the correctness of PSMTs' definition of the mathematical terms in their video-lesson presentations by taking into account the works of Leikin and Zazkis' [30] and Borasi [7]. In this study, the research questions are the following:

1. How prevalent were the errors in definitions and what were the errors commonly committed?

2. How prevalent are the errors when all the definitions are grouped by subject area?

Findings of the study would provide necessary baseline data to assess the need to implement an intervention program that would help develop and improve PSMTs' skill in crafting and using good definitions of mathematical terms.

2 Background Literature

2.1 Mathematics Content Knowledge

Teachers' knowledge of fundamental mathematics definitions plays an essential role in mathematics teaching and learning [3, 5, 11, 13, 16]. It is considered as a core element of mathematics teachers' content knowledge (CK) [4, 12, 30, 39] which refers to the teachers' knowledge of fundamental mathematics definitions, concepts, and procedures towards student learning [4, 37, 46]. In recent years, contribution of teachers' mathematical CK has received increased attention from scholars, teacher educators, policymakers and professional organizations worldwide [3, 5, 12, 19, 27, 36]. Mathematical definitions, as an important part of CK, introduce mathematical objects, concepts and ideas, required for understanding mathematical processes and are necessary elements in problem solving, proving, and communication [46]. Once they are determined in a curriculum, they affect the approach to teaching, the learning sequence, and the set of theorems and proofs [30].

2.2 Concept Definition and Concept Image

Students' acquisition of fundamental mathematics definitions can be understood by the interrelationship between concept definition and concept image. How a mathematical concept definition is presented to students shapes its concept image [42, 45]. Concept definition is the verbal explanation of the student's concept image by the student [42]. In other words, concepts are mainly obtained by means of definitions. Concept image is defined as all the cognitive structure in the individual's mind that is associated with a given concept [42] which covers all mental pictures (pictorial, symbolic, and others), all mental attributes (conscious or unconscious), and associated processes [43]. According to Vinner [45], understanding of a concept definition or conceptual understanding happens when one has formed or acquired a concept image of a concept definition which is only possible when certain meaning has been attached with the words in the definition [45]. Thus, knowing the definition by rote memorization does not guarantee formation of concept image of the definition. Due to individual differences in sensory associations, one's concept image may vary from the other, and student's experiences related to examples of a concept in school, textbooks, or other contexts play a significant role in the formation of a concept image [43]. This means that student's concept image can also develop into misconceptions depending on their experiences with examples and non-examples of the concept. With lack of experience and opportunity to capture meaning, there is a possibility that the student cannot articulate entirely his concept image in words, that is, he cannot come up with concept definition, or the student's concept image might be connected to erroneous examples. This may lead to student unease with the concept in general.

It is therefore imperative for mathematics teachers to develop a teaching strategy to help students improve their conceptual understanding of the mathematical concepts. Teachers should know at what stage of students' mathematical learning experience to introduce a particular definition of a term and when to explain the equivalence of the two or more definitions of a term to the students. For example, when the concept "function" is defined in two ways: (1) as a relation in which every element of the domain is paired with exactly one element of the range, and (2) as f which is a rule that assigns to each element x in a set D exactly one element called $f(x)$, in a set E [11]. The former definition has a concept image which is connected primarily to a set of ordered pairs. A teacher usually introduces this kind of function definition in pre-calculus subjects. The latter definition on the other hand has a concept image which is connected to the algebraic expression $f(x)$ where x is called an x-intercept when $f(x) = 0$, and $f(x)$ is called a y-intercept when $x = 0$. Teacher usually introduces this kind of function definition in a Calculus subject where students already have a strong foundation about functions during pre-calculus. Another example which is illustrated in the studies of Johnson et al. [22] and Morgan [34] is when to introduce each of the two definitions of a circle: the metric definition (focusing on the idea that all points on a circle are equidistant from a given center), and the analytic definition (expressed in the form of an equation such as $(x\check{\ }a)^2 + (x\check{\ }b)^2 = r^2$).

2.3 PSMTs' Mathematical Definitions

A number of empirical studies have investigated pre-service and in-service mathematics teachers' choices, use and understanding of mathematical definitions [5, 11, 13, 14, 22, 27, 30]. Findings of these studies suggest teachers' deficiency in their knowledge of what a good mathematics definition is all about. Unfortunately, many prospective teachers still have difficulty understanding the mathematical content that they have to teach because they have weak concept images. In Leikin and Zazkis [30], teacher-generated definitions of concepts from different mathematics content areas were analyzed focusing on the correctness and richness of the definitions. They found that pre-service mathematics teachers' incorrect examples of definitions lack necessary and sufficient conditions. According to them, the result has to do with pre-service teachers' lack of understanding of the specific concept and its critical features, or lack of understanding of the concept of definition itself. This result was also evident in Morgan [34] where pre-service mathematics teachers' difficulty of creating definitions was due to poor concept images for the concept being defined, and poor concept images for other concepts related to the concept being defined. In the study of Chesler [13], PSMTs' responses to three tasks that asked them to choose, use, evaluate, and analyze definitions were analyzed. The results showed that many PSMTs have difficulty with choosing, interpreting, comparing and evaluating definitions. It was indicative of their lack of flexibility in terms of word choices, lack of flexibility and expertise in interpreting and using mathematics definitions, and weak knowledge about definitions to correctly assess the possible equivalence of two or more definitions. The study of Johnson et al. [22], revealed a teacher's inability to distinguish between a description and a definition, and their use of unprecise terms or everyday registers in their definitions. Similarly, analysis of pre-service teachers' video-recorded lessons in Lane et al. [27] revealed misuse of certain basic mathematical terms, use of unprecise terms that adhered more to everyday register or everyday speech, and inability to explain some mathematical terms they referenced in their lesson.

A number of studies recommended some possible solutions to address the difficulties and misconceptions of pre-service and in-service teachers in "defining" mathematical terms. One is the need to review definitions in textbooks since many instructors depend on definitions from textbooks [5, 11] although some do not necessarily follow textbook authors' method or level of detail [11]. The study in Berger [5] discovered textbooks with definitions that are unnecessary or inappropriate for a certain level of mathematics curriculum. Another solution is the need for instructors to be mindful with the kind of definitions they present [5], and the ways they present their definitions in classes. According to Berger [5], instructors should make sure that students (1) acquire a clear and complete formal definition of a term or concept, (2) are given opportunities to create their own informal definitions before the instructor presents the formal version, (3) are allowed to find and compare multiple versions of a definition, and (4) are encouraged to create graphical interpretation of a definition.

2.4 Conceptualizations and Characterizations of a Good Mathematical Definition

A number of scholars had done extensive work on conceptualization of mathematical definitions [7, 30, 34]. Borasi [7, pp. 17 – 18] identified five characteristics of a good definition:

1. Precision in terminology. All the terms employed in the definition should have been previously defined, unless they are one of the few undefined terms assumed as a starting point in the axiomatic system one is working with.

2. Isolation of the concept. All instances of the concept must meet all the requirements stated in its definition, while a non-instance will not satisfy at least one of them. There should be no counterexample (an example that would disprove your definition) and there should be no missing restriction.

3. Non-contradicting. The properties stated in the definition should be able to coexist.

4. Essentiality of the concept. Only terms and properties that are strictly necessary to distinguish the concept in question from others should be explicitly mentioned in the definition. There should be no unnecessary or wrong property.

5. Non-circularity. The definition should not use the term it is trying to define.

Borasi [7] as cited in Morgan [34, p. 106] categorized the 5 characteristics of a good definition into two criteria for a good definition:

1. Definition should allow us to discriminate between instances and non-instances of the concept with certainty, consistency, and efficiency (by simply checking whether a potential candidate satisfies all the properties stated in the definition); and,

2. Definition should 'capture' and synthesize the mathematical essence of the concept (all the properties belonging to the concept should be logically derivable from those included in its definition).

A number of studies highlighted Borasi's [7] characterizations of a mathematics definition. For example, Morgan [34] considered Borasi's [7] characterizations of a mathematics definition and highlighted the arbitrariness of a mathematical definition of a term. According to Morgan [34], a single object (that is a mathematical term) may be defined in several logically equivalent ways and such alternative definitions facilitate the generation of different types of theorems, proofs and solution methods. Examples are the multiple definitions of a "function" and of a "circle". Leikin and Zazkis [30] developed a framework for analyzing teacher-generated definitions. According to the framework, definitions can be evaluated in terms of accessibility, correctness (appropriateness), richness, and generality/concreteness. The focus in the present study is the evaluation of the correctness of the mathematics definitions of PSMTs in their video-lesson presentations. Leikin and Zazkis [30] categorized correctness of mathematical definitions as (1) *appropriate and rigorous*, (2) *appropriate but not rigorous*, and (3) *inappropriate*. *Appropriate and rigorous definitions* include necessary and sufficient conditions of the defined concept as well as accurate mathematical terminology and symbols, and are usually minimal. *Appropriate but not rigorous definitions* omit some constraints, or used imprecise terminology and not minimal. *Inappropriate definitions* include those that lack necessary or sufficient conditions. They include those definitions that present only specific instances, only name the concept, or provided ill-defined conditions. In Leikin and Winicki-Landman [29], mathematical definitions were described as follows: (1) it presents the name of the concept, and this term (name) appears only once in this statement; (2) it establishes necessary and sufficient conditions for the concept; (3) in defining the new concept, only previously defined concepts or basic concepts (or undefined concepts) may be used; (4) the set of conditions must be minimal. Buiza [9] asserted that a good definition must satisfy two conditions: "First, it classifies the term being defined under a category where it belongs. Then it specifies the characteristics that distinguish the term from the other terms in that category" [9, p.4]. Figure 1 illustrates the conditions for a good definition.

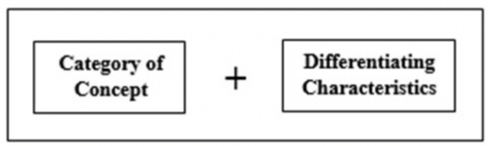

Figure 1: Two Conditions for a Good Definition

For example, "A hexagon is a six-sided polygon" is a definition of hexagon obtained from one of the PSMT video-lesson presentations in this present study. In the definition it is clear that the term "hexagon" belongs to category "polygon" but among the polygons, its differentiating characteristics is that it is "six-sided". Buiza's [9] conditions for a good definition resonates with Borasi's [7] two criteria for a good definition.

2.5 Previous Studies using Leikin and Zazkis' Categories of Correctness of Mathematical Definitions

Leikin and Zazkis' analytical framework has been used in a number of studies to examine the quality of pre-service teachers' mathematical definitions. In Leikin, and Zazkis [30], Leikin and Zaskis' analytical framework was used to examine pre-service teachers' generated examples of definitions in geometry, algebra, and calculus. Ulusoy [43] analyzed prospective elementary mathematics teachers' concept image and concept definitions of triangles through a defining task and an example generation task using Leikin and Zaskis' analytical framework. Also, Kubar and Cakiroglu [25] used Leikin and Zazkis' correctness criterion to examine prospective elementary teachers' understanding about integer concept. The authors analyzed the appropriateness of the teacher-generated definitions. Results showed that prospective teachers have difficulty in determining correctness of definition. In Aktas [1], Leikin and Zazkis' correctness criterion was used to examine the appropriateness of high school students' definitions for parallelograms. The study revealed that most students gave inappropriate definitions which lacked necessary or sufficient properties. In the study of Zaskis and Leikin [47], Leikin and Zazkis' analytical framework was used to explore PSMTs' example definitions of a square in terms of accessibility, correctness, richness and generality. In general, Leikin and Zazkis' analytical framework has shed some light on how mathematics definitions should be crafted by pre-service mathematics teachers.

3 Methodology

3.1 Research Design

In the present study, the authors qualitatively and quantitatively analyzed the definitions of mathematical terms PSMTs provided in their video-lesson presentations. In this study, an analytical framework was developed based on the works of Leikin and Zazkis [30] and Borasi [7].

3.2 Participants and Setting

The participants of the study were PSMTs (N=67, 27 males, 40 females, 19 to 24 years old) who were third year Bachelor of Science in Secondary Education major in Mathematics (BSEd Mathematics) students during the conduct of the study. BSEd Mathematics is a four-year degree program in a Philippine university, at the end of which PSMTs will be qualified to take the Licensure Examination for Teachers (LET) facilitated by the Philippine Regulatory Commission (PRC). If they pass the LET, they become licensed and qualified secondary level (Grades 7 to 12) mathematics teachers in the country. Prior to the study, the PSMTs already completed several mathematics content subjects (College Algebra, Plane and Solid Geometry, Plane and Spherical Trigonometry, Elementary Statistics, Set Theory and Logic, Number Theory, Differential Calculus and Integral Calculus) and pedagogy subjects (Assessment of Student Learning and Educational Technology).

3.3 Data-Gathering Procedure

There were three sections in third year BSEd Mathematics. As part of the requirements of the PSMTs in their mathematics pedagogy subject during the first semester of school year 2020-2021, their instructor asked all PSMTs in each of the three sections to divide into groups of 5 or 6 members. Each group came up with a 15-to-60-minute video-lesson presentation in a peer-teaching setting. There was a total of 13 PSMT groups. Each group was given 3 weeks for the preparation. The instructor is a veteran mathematics teacher who has taught mathematics for at least 15 years in different levels (high school and college levels) and is a doctoral degree holder in mathematics education. Each video-lesson presentation has one lesson. A lesson was a topic in algebra, geometry, number theory or statistics. Each lesson was aligned with the Philippine secondary mathematics curriculum as reflected in the K-12 Curriculum Guide Mathematics (K-12 Curriculum Guide Mathematics, 2016). Each lesson was divided into a sequence of four episodes: (1) review of past lesson, (2) ice breaker, (3) presentation of new lesson and (4) assessment. The choice of lesson and the assigning of PSMTs in each of the episodes were left to each group's discretion. In this study, the analysis focused only on the definitions of mathematical terms seen or mentioned during the review and new-lesson-presentation episodes of the lesson in the video-lesson presentation. Table 1 above shows the different PSMT groups, chosen lesson per group, and the number of definitions each group provided in their video-lesson presentations.

There was a total of 109 definitions from a total of 90 different terms defined across groups. There were terms which were defined more than once within a group. For example, in G4, two members, each, gave two different definitions of the term "polynomial" in their lesson (first, during review/motivation episode and second, during lesson proper). There were also some terms defined by two groups lessons. For example, G1 and G4 presented a definition of the term "polynomial" in their lesson.

Table 1: PSMT Groups, Lessons and Number of Definitions

PSMT Groups	Lesson	No. of Definitions
G1	Classifying and Evaluating Polynomials	11
G2	Measures of Central Tendency	9
G3	Linear Equation in One Variable	9
G4	Polynomials	10
G5	Regular Polygons	10
G6	Lines and Subsets of Lines	10
G7	Real Number System	13
G8	Laws of Exponents	3
G9	Quadratic Equation in One Variable	3
G10	Linear Equation in One Variable	2
G11	Direct Proof	6
G12	Linear Equation	5
G13	Basic Operations on Sets	18
Total		109

3.4 Data Analysis

Frequencies and percentages were used to describe the number and proportion of definitions in each category of correctness of the definitions given by PSMTs during their peer-teaching video-recorded lesson presentations All the 109 definitions were recorded and analyzed. The authors developed a framework to analyze the correctness of PSMTs' definition of the mathematical terms in their video-lesson presentations based on the works of Leikin and Zazkis [30] and Borasi [7]. In this study, the PSMT definitions were categorized either as *good definition*, *satisfactory definition*, or *weak definition*. Using deductive content analysis, the authors developed a deductive coding scheme based on the convention of Bikner-Ahsbahs et al. [6]. *Good definitions* were coded as A. Those which were considered as *satisfactory definitions* were those which lacked minor details but did not commit contradiction or misconception. They were coded as omission of minor conditions (B1), use of imprecise terminology (B2), not minimal (B3), and minor grammatical error (B4). *Weak definitions* (or unsatisfactory definitions) were those which lacked necessary or sufficient condition, created confusion or misunderstanding, contradiction or misconception. They were coded as non-isolation of concept (C1), contradictory condition (C2), circularity (C3), representing specific instance or naming of concept only (C4), and use of less precise words (C5). In general, a PSMT definition can only belong to one of the three categories of correctness. Tables 2 and 3 shows the coding manual which contains the coding rules, and anchor examples. In coding, a PSMT definition can only belong to one of the three categories of correctness. If in case a PSMT definition possesses two characteristics from two different categories of correctness, the PSMT definition will belong to the lower category of correctness. A PSMT definition may be given one or more different codes within the same category of correctness where it belongs as long as the coding rules apply.

Intrarater reliability was used following the convention of Gwet [17] to measure self-consistency in the categorization of the PSMTs' mathematical definitions. The first author categorized the PSMT's definitions on two occasions (first replication and second replication) separated by a one-month interval between the two replications. The choice of one-month interval was based on Bottari et al. [8] which is enough to reduce the effect of author's memory of categorization during the first replication. Consequently, the kappa coefficient that measures intrarater reliability of the categorization of definitions into three nonoverlapping categories, namely, good, satisfactory and weak was 0.91 which indicates a near perfect agreement in the author's categorization on two occasions (see Appendix A for the categorization matrix). When categorization was based on the eight codes (A to C5), the

Table 2: Categories of Correctness of Pre-service Secondary Mathematics Teachers' (PSMTs') Definitions of Mathematics Terms

Category of Correctness	**Code**	**Characteristics**	**Coding Rules** (A PSMT definition is given the code if . . .)	**Anchor Examples** (PSMT Definitions extracted from PSMTs' Video-Lesson Presentations)	**Reason(s)**
Good Definition	A	Inclusion of necessary and Sufficient Conditions	category of the concept and differentiating characteristics are clearly stated.	Hexagon is a six-sided polygon.	All characteristics of an appropriate and rigorous definition were observed
		Use of Accurate Math Terminology and Symbols	all the terms employed in the definition have been previously defined, unless they are one of the few undefined terms.		
		Minimality	non-necessary conditions and words are avoided.		
Satisfactory Definition	B1	Omission of Minor Conditions	there is missing a condition or constraint due to carelessness or lack attentiveness but no contradiction in the definition is being committed.	A linear equation in one variable is an equation in the form $ax + b = 0$.	Minor constraint "where a and b are real numbers and a is not equal to zero" was omitted which may be due to carelessness.
	B2	Use of correct term but not previously defined	a term which is not previously defined is used in the PSMT definition.	Real number is composed of the rational and irrational numbers.	"The set of real numbers" is the precise term to be defined.
	B3	there is an unnecessary condition or if the definition seems to be too wordy.	A solution is a value such as when you replace the variable with it, it makes the equation true.	The definition is too wordy.	
	B4	Lacking rigor in the use of language	there is minor grammatical issue which is quite distracting.	Equation is a statement that the values of two mathematical expressions are equal.	There is a minor grammatical error in "statement that the values" which is more distracting than confusing.

Table 3: Categories of Correctness of Pre-service Secondary Mathematics Teachers' (PSMTs') Definitions of Mathematics Terms

Category of Correctness	**Code**	**Characteristics**	**Coding Rules** (A PSMT definition is given the code if . . .)	**Anchor Examples** (PSMT Definitions extracted from PSMTs' Video-Lesson Presentations)	**Reason(s)**
	C1	Non-isolation of concept	there is an instance of the concept that does not satisfy one of the requirements stated in the PSMT definition, or there is a non-instance of the concept that can satisfy all the requirements stated in the PSMT definition.	A polynomial is an algebraic expression which consist of variables and coefficients.	The concept is not isolated. One may think that $(3x/y) + 2w = 5$ is a polynomial based on the given definition.
	C2	Contradictory Conditions	there are two properties or conditions stated in the PSMT definition that are impossible to coexist.	A polynomial is a sum and difference of polynomial terms.	A polynomial cannot be a sum and difference of terms at the same time.
Weak Definition	C3	Circularity	the PSMT definition uses the term it is trying to define.	A polynomial is a sum and difference of polynomial terms.	The term "polynomial" is used in the definition.
	C4	Representing specific instance or naming of concept only	the PSMT definition is simply describing or listing instead of defining; giving only specific examples, names or illustrations.	Algebraic expression consists of variables and coefficients.	The definition is confusing. The definition merely gives description or states specific instance of a term.
	C5	Use of less precise or confusing words	a mathematically less appropriate language is used which led to confusion or misunderstanding, or the PSMT definition is confusing due to major grammatical issues.	A ray is a point and extends forever in one direction.	The phrase "that extends forever" is imprecise and can lead to misconception.

Table 4: Value of Kappa Coefficient k and the Corresponding Interpretation

Value of Kappa Coefficient k	Interpretation
$0.81 \leq k < 1.00$	Almost Perfect Agreement
$0.61 \leq k \leq 0.80$	Substantial Agreement
$0.41 \leq k \leq 0.60$	Moderate Agreement
$0.21 \leq k \leq 0.40$	Fair Agreement
$0.00 \leq k \leq 0.20$	Slight Agreement
$k < 0$	Poor Agreement

kappa coefficient was 0.68 which indicates a substantial agreement. The latter's lower value of the kappa coefficient was due to the complexity of categorization when eight categories (or codes) were considered. In this categorization, a definition can be given one or more codes if classified as a satisfactory or weak definition. For example, the author classified definition of leading coefficient which was "When a polynomial is written with decreasing exponents, the coefficient of the first term is the leading coefficient" as a weak definition and gave it three codes, namely, C3, C4 and C5. This was due to issues of circularity, use only of specific instance of the definition, and use of less precise words, being evident in the given definition. Despite the complexity, the values of kappa coefficients in the present study were nevertheless within the acceptable range [8, 26]. Table 4 shows the value of kappa coefficient k and the corresponding interpretation based on Landis and Koch [26], as cited in Bottari et al. [8].

After the intrarater reliability was determined, any disagreements in the categorization of definitions during the first and the second replications were identified and reviewed. The categorixations were then finalized.

4 Results

4.1 Correctness of Definition of Mathematical Terms

The distribution of correctness is shown in Figure 2. The PSMTs gave 23 good definitions or 21% of all definitions. There were 29 satisfactory definitions (or 27% of all definitions) and 57 weak definitions (or 52% of all definitions).

Figure 2: Correctness of Definitions of Mathematical Terms

4.1.1 Satisfactory Definitions

Figure 3 shows the distribution of 29 satisfactory definitions committed by the PSMTs.

Figure 3: Distribution of 29 Satisfactory Definitions

Analysis revealed 17 (or 59%) of the 29 satisfactory definitions were due to omission of some minor constraints (or B1). Eight out of 13 PSMT groups committed this kind of mistake in some of their definitions (G1, G2, G3, G7, G8, G9, G11 and G13). For example, G10 defined "*odd number*" this way: "*A number n is odd if and only if there exists k element of Z s.t.* $n = 2k+1$." Here, the symbol Z representing the set of integers was not mentioned. Another example was a definition of "*linear equation in one variable*" given by G3 which was "*may take the form* $ax + b = 0$ *and are solved using basic algebraic operations.*" Here, restrictions on a, b, and x were not stated although the definition was understandable. Seven (or 24%) of 29 of the satisfactory definitions were imprecise because they contain terms that were not previously defined (or B2). Five out of 13 PSMT groups committed this mistake (G2, G3, G4, G5, and G11). For example, in the definition of "*statistics*" given in G2 which was "*form of mathematical analysis that uses quantified models, representation and synopsis for a given set of experimental data or real-life studies.*" In this definition, a number of technical terms like models, synopsis and experimental data were not explained. Technical terms used in the definition should also be defined explicitly in a way that relates to the context of students. Moreover, six (or 21%) out of 29 satisfactory definitions were not minimal (or B3). Four PSMT groups committed this mistake (G5, G9, G12, and G13). For example, G3 defined "*solution*" as "*a value, such that, when you replace the variable with it, it makes the equation true*". A possible minimal version could be *"A solution is a value that makes the equation true"*. Furthermore, there was just one (or 3%) out of 29 satisfactory definitions due to minor errors in grammar (or B4). Only one PSMT in G10. The PSMT gave the definition of "*equation*" as "*a statement that the values of two mathematical expressions are equal*".

4.1.2 Weak Definitions

Out of 109 definitions, 57 were categorized as weak definitions. They were further categorized into 5 characteristics (see Fig. 4).

Seventeen (or 30%) of 57 definitions provided by the PSMTs were weak because they were not isolated (or C1). Nine out of 13 PSMT groups committed this mistake (G1, G3, G4, G7, G8, G9, G10, G12 and G13). As an example, G3 defined "*linear equation*" as "*an equation of a straight line, written in one variable. The only power of the variable is 1.*" Here, the definition could mislead students to believe that a linear equation can only have one variable. In fact, a linear equation can have one or more variables like $x - 6 = 0$, $0.5x + y = 10$ or $3x - 2y + z = 4$. Another example was a definition of "*polynomial*" given by G4 which was "*algebraic expression which consist of variables and coefficients.*" Here,

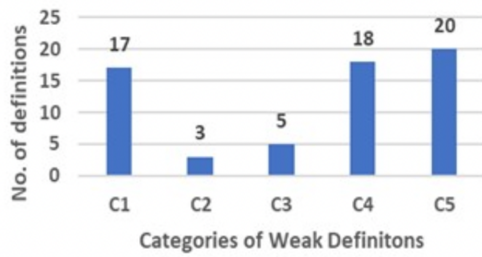

Figure 4: Distribution of Weak Definitions

we can show a non-instance of the concept image of the term polynomial, that can satisfy the given definition, let us say, $\frac{3x}{y} + 2w = 5$ which is an algebraic expression consisting of variables and coefficients but is not actually a polynomial.

Three (or 5%) of 57 weak definitions provided contradicting properties (or C2). Only two of 13 PSMT groups committed this mistake (G4 and G6). An example was the definition of "*midpoint*" given by G6 which was "*a point that divides a line or line segment into two congruent parts*". Here, the definition seems to suggest that a line can have a midpoint which is a contradiction since a line has no midpoint.

Five (or 9%) of 57 weak definitions committed circularity (or C3). That is, the term being defined was being used in the definition. Four PSMT groups committed this mistake (G1, G4, G5 and G11). For example, G4 defined "*degree of a polynomial*" as "*in any polynomial written in descending order, the leading term would tell us the degree of the whole polynomial*". Another example is a definition of "*polynomial*" as a "*sum and difference of polynomial terms*".

Eighteen (or 32%) of the 57 weak definitions represented only specific instances of the terminology or name only the term (or C4). Nine out of 13 PSMT groups committed this mistake (G1, G2, G3, G4, G5, G10, G11, G12 and G13). For example, a PSMT in G6 defined "*ray*" as "*In ray CD, the first letter C is the first endpoint and the second letter D is the second endpoint.*" Another example was given by a PSMT in G4 which was the definition of "*constant of a polynomial*" as "*usually seen at the tail end of the expression*". Furthermore, G1 defined "*numerical coefficient*" as "*the number in the algebraic term*".

Twenty (or 35%) of 57 weak definitions used less precise or confusing terms (or C7). Nine out of 13 PSMT groups committed this kind of mistake in some of their definitions (G1, G2, G4, G5, G6, G7, G9, G12 and G13). For example, G2 defined "*median*" as "*the midpoint of the array. It is arranged in decreasing or increasing order. The median will be either a specific value or will fall between two values.*" Here, median being arranged in decreasing or increasing order might mislead students to believe that a median can be two or more values that can be arranged in the order. A second example was the definition of "*ray*" provided in G6 which was "*It is a portion of a line which has one endpoint and extends forever in one direction.*" Here, the definition could mislead students to believe that a point can extend. The phrase "*extends forever in one direction*" is an everyday English phrase which is not mathematically precise and can lead to misunderstanding or misconception. A third example was a definition of "*prime number*" given in G7 which was "*the one which has exactly two factors, which mean, it can be divided by '1' and itself.*" Here the prime number was broadly categorized as "*the one*" which is an obscure category. If it was instead "*natural number*", the given definition would be clearer.

4.2 Correctness of Definition of Mathematical Terms by Subject Area

All 109 definitions provided by PSMTs were categorized in 4 subject areas, namely, algebra, geometry, number theory, and statistics, where each of them belongs. Nine PSMT groups (G1, G3, G4, G7, G8, G9, G10, and G12) developed video-lesson presentations whose lessons were under algebra. Two groups (G5 and G6) developed video-lesson presentations whose lessons were under geometry. Two groups (G11 and G13) developed video-lesson presentations whose lessons were under number theory. Only one group (G2) developed a video-lesson presentation whose lesson was under statistics. There was a total of 56 definitions under algebra, 20 definitions under geometry, 24 definitions under number theory, and 9 definitions under statistics. The distribution of correctness of definitions of mathematical terms by subject area is shown in Figure 5.

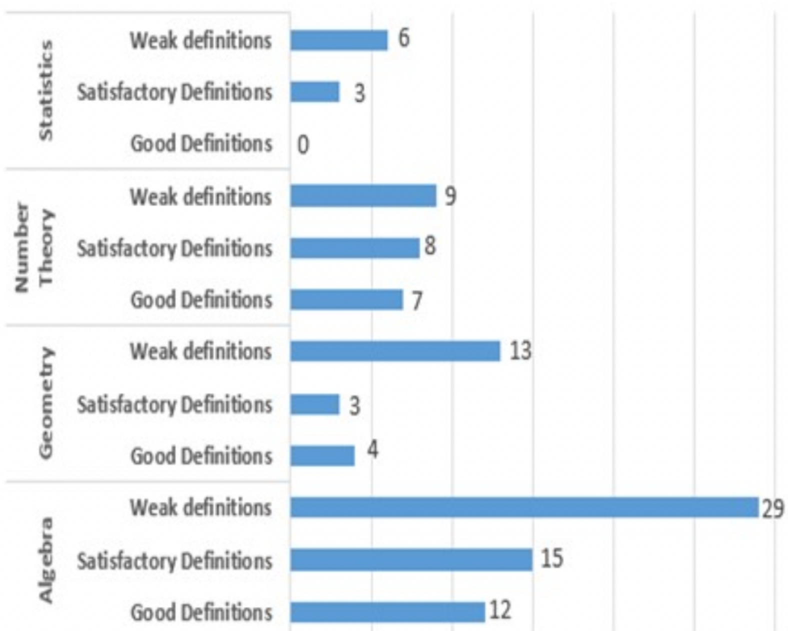

Figure 5: Distribution of Definitions by Subject Area

In each of the 3 subject areas, namely, Algebra, Geometry and Statistics (refer to figure 6), it can be seen that more than 50% of the definitions given by the PSMTs were considered as weak definitions. There were 29 (or 52%) weak definitions out of the 56 definitions in algebra. There were thirteen (or 65%) weak definitions out of 20 definitions in geometry. In statistics, there were six (or 67%) weak definitions out of 9 definitions. Although the number of weak definitions in number theory was only 9 (or 38%) out of 24 definitions, it was still the highest in the subject area.

Figure 6 shows the number of satisfactory definitions that committed B1, B2, B3 and B4 mistakes per subject area. In algebra, ten (or 67%) of 15 satisfactory definitions were due to omission of minor constrains (B1). In geometry, three (or 100%) of three satisfactory definitions were due to minimality issue (B3. In number theory, six (or 75%) of eight satisfactory definitions were due to B1. In statistics, two (or 67%) of three satisfactory definitions used imprecise terminology (B2).

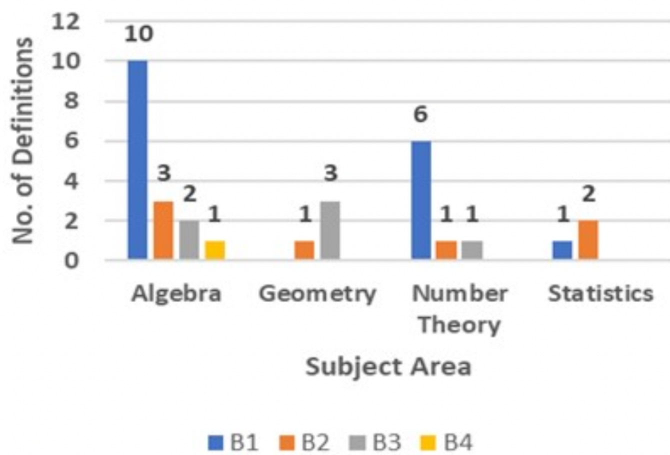

Figure 6: Number of Satisfactory Definitions per Subject Area that Committed B1, B2, B3, and B4.

Figure 7 shows the number of weak definitions that committed C1, C2, C3, C4 and C5 mistakes per subject area. In algebra, 16 (or 55%) of 29 weak definitions are due to non-isolation of concepts (C1). In geometry, nine (or 69%) of 13 weak definitions were due to use of less precise or confusing words (C5). In number theory, five (or 56%) of nine weak definitions represented specific instances only (C4). Lastly, in statistics, five (or 83%) of six weak definitions were due to C4.

5 Discussion

In the present study, PSMTs appeared to lack the precision needed in stating definitions of mathematical terms. Most of the PSMTs believed that their definitions in their video-lesson presentations were correct. They argued that they checked their sources. However, some confessed they felt unsure of the correctness of their definitions but did not mind since it came from a source or it was agreed by their respective PSMT groups. PSMTs' inability to recognize erroneous mathematical definitions in their video-lesson presentations may be due to their (1) weak knowledge about the characteristics of a good mathematics definition, and (2) lack rigor in the use of English language to clearly express the intended meaning of their definitions.

One possible reason for the predicament may be due to PSMTs' overreliance on definitions that were readily made or provided by their teacher, textbooks, lecture notes, internet and other sources without them being given opportunities to verify the correctness. Studies of Tall and Vinner [42], Vinner [45] and Capaldi [11] reminded mathematics teachers to be mindful of the ways they present their definitions to students. According to them, how mathematical terms or concepts are presented to students affects the development of students' concept images of those terms. Knowledge of mathematical terms is an integral

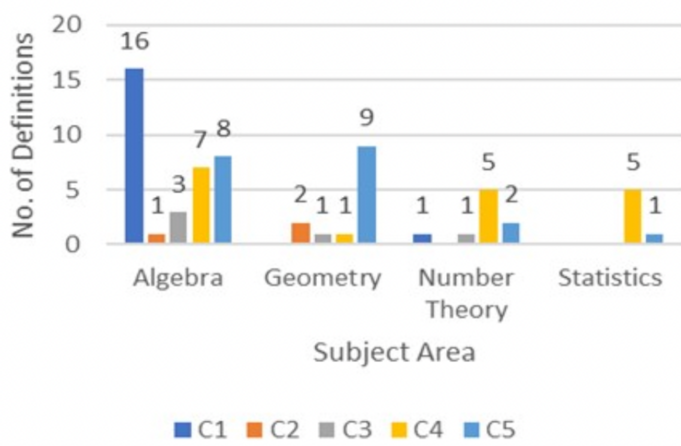

Figure 7: Number of Weak Definitions per Subject Area that Committed C1, C2, C3,C4,and C5.

part of students' knowledge structure which largely influences students' thinking processes [42, 45].

In the present study, the PSMTs' weak knowledge of a good definition of a mathematical term was evident in their inability to distinguish between description and definition which is the same finding as in the study of Johnson et al. [22]. In their work, Jonson et al. [22] observed teachers frequently replaced the role of definition by description or exemplification. They warned that mere describing or providing examples only instead of giving the definition, may run the risk of not being able to provide sufficient information to discriminate between instances and non-instances of the concept with certainty, consistency, and efficiency, which may lead to misconception. In the study of Dickerson and Pitman [14], teacher participants felt the need to include key examples when writing definitions to help develop clear concept image of the term under investigation. Thus, in presenting a definition, the definition should be precise and it should be followed with the presentation of key examples. In the present study, many mathematical definitions were presented by the PSMTs' with examples in their video-lesson presentations. There were many instances, though, where PSMTs presented weak definitions of mathematical terms but provided correct choices of examples. These instances may be attributed to what Chesler [13] referred to as conflicting cognitive schemes for PSMTs' concept image and concept definition in each of those mathematical terms. This conflict is also manifested in some PSMTs' inability to explain clearly some of the definitions they presented in the video lessons in the present study.

In the present study, PSMTs lack of knowledge in choosing or crafting good mathematical definitions is similar with the result of the study of Leikin and Zazkis [30], and Chesler [13] pointing out PSMTs' lack of knowledge of good definitions may have to do with lack of understanding of the concept of definition itself. The present study's findings regarding PSMTs' weak definitions corroborated as well with the results of the studies of Lane et al. [27] and Johnson et al. [22] that PSMTs revealed misuse of certain basic mathematical terms, use of unprecise terms that adhered more to everyday register or everyday speech which affected the clarity of the meaning which the term intends to employ.

Another reason why PSMTs in the present study fall short in attending to the precision of their mathematical definitions may largely be due to their lack of rigor in the use of English language in expressing the precise meaning of their definitions. When the mathematical definitions are to be stated in English language which is not the PSMTs' first language or natural language spoken at home (L1), the act of defining can pose a serious challenge. A number of studies argued low English proficient students can improve their English as a second (L2) by letting them use first their L1 and gradually letting them use L2 [32, 44]. In the case of multilingual learners, studies suggested the use of translanguaging by encouraging the students to express their thoughts by allowing them to use all their languages repertoire [31, 41].

In the present study, PSMTs' lack of knowledge regarding qualities of good mathematical definitions could be a reason why none of them considered creating their own definitions for their video-lesson presentations and instead opted for "ready-made" definitions. A PSMT argued that as a PSMT, they do not have the "authority" yet to create their own definitions. Capaldi [11] suggested that teachers should give students opportunities to (1) acquire a clear and complete formal definition of a term or concept, (2) let students create their own informal definitions before the instructor presents the formal version, (3) find and compare multiple versions of a definition, and (4) create a graphical interpretation of a definition.

Leikin and Zazkis [30] argued that mathematics teachers should exhibit knowledge of various definitions of a mathematical term or concept, the differences of these definitions at various levels of mathematics curriculum, and the differences in corresponding learning sequences for their choice of definition of a mathematical concept. For example, they should know the differences in the definitions of a "function" at various junctures of the mathematics curriculum. When speaking of the concept "circle", teachers should know its different equivalent definitions in Euclidean geometry, analytic geometry, and algebra [30].

In the present study, it was still unknown as to whether PSMTs' mathematical CK has played a role in PSMTs' deficiency in assessing the correctness of the mathematical definitions in their video-lesson presentations. Leikin and Zazkis [30] posited that PSMTs' lack of knowledge of good definitions may have to do with lack of understanding of the mathematical CK. Future studies should be conducted to gather empirical evidences regarding the relationship between PSMTs' mathematical CK and their knowledge about mathematical definitions.

Future studies can also look into how precision of mathematical definitions is taken into account during the preparation of student-created video-lesson presentations. Attending to precision of definitions should be one of the many key considerations in creating effective mathematics video-lessons for students. In the present study, PSMTs' lack of precision in stating definitions of mathematical terms in their video-lesson presentations may be attributed to little attention given to them during video-lesson preparation. Other things are usually given more attention in order to come up with a successful video-lesson presentation. These include high-quality appearance, delivery, voice, background music, problem-solving process, and detailed explanation of the applications of mathematical concepts in the real world (Loch et al., 2016).

6 Conclusion

With the rising popularity of student-created video lessons in mathematics teaching and learning, it is necessary to pay attention to how mathematical ideas are being communicated in the video lessons. Attending to precision in crafting definitions should be one of the many key considerations in creating effective mathematics video-lessons for students. PSMTs' ability to craft good mathematical definitions can positively influence their ability to explain their mathematical CK accurately and comprehensively whose benefits can extend to their performance in problem-solving and on standardized assessments. Furthermore, attending to precision in PSMTs' mathematical definitions can provide a glimpse of their readiness to teach mathematics at the secondary or high school level.

In this paper, the authors have taken into account the correctness of the definitions found in the PSMTs' video-lesson presentations. In this study the authors developed an analytical framework based on the works of Leikin and Zazkis [30] and Borasi [7] to analyze the quality of PSMT's definitions. Analysis revealed PSMTs' lack of precision needed in stating definitions of mathematical terms. This could be attributed to PSMTs' lack of knowledge about what counts as a good definition of a mathematical term, and lack rigor in the use of the English language to clearly express the precise meaning of their definitions.

The research findings have implications for how mathematics teachers assess the definition of mathematical terms they plan to present in their lessons. They can use the analytical framework proposed in this present study as an effective lens through which to determine the quality of their definitions. For PSMTs, the analytical framework can potentially enhance their ability to communicate mathematical concepts and ideas precisely and clearly, which is an important skill for them to develop in order to be able to teach mathematics effectively.

Therefore, the authors recommend more exposure of the PSMTs to activities that develop their skill in defining. Moreover, follow-up studies should be conducted that would further guide mathematics educators in designing intervention programs that gear towards development and improvement of PSMTs' skills in crafting good mathematical definitions.

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Appendix A

Distribution of 109 Mathematical Definitions by Three Categories of Correctness and Replication Number

First Replication	**Second Replication**			**Total**
	Good Definitions	Satisfactory Definitions	Poor Definitions	
Good Definitions	24	0	0	24
Satisfactory Definitions	2	24	2	28
Poor Definitions	0	2	55	57
Total	26	26	57	109